

Discrete Structure

TUTORIAL MANUAL

SUBJECT:

DISCRETE STRUCTURE

CLASS:

(Computer / I.T)

Semester III

Discrete Structure

<i>Sr No.</i>	<i>Name</i>	<i>Page No</i>
1	SET THEORY	3,4
2	SET THEORY AND LOGIC	5,6
3	RELATIONS	7,8,9
4	RELATIONS	10,11
5	LATTICES	12,13
6	TYPES OF LATTICES	14,15
7	FUNCTION	16
8	GROUP	17,18
9	GRAPHS	19,20, 21
10	RINGS AND FIELDS	22,23
11	ENCODING	24,25
12	DECODING,PIGEONHOLE PRINCIPLE	26,27

TUTORIAL NO 1

TITLE :-SET THEORY

OBJECTIVE:-To study set theory.

REFERENCE:-

1. Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
2. 2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
3. Previous Questions Papers.

PREREQUISITE:-

- 1>Sets, Venn diagram, set membership table
- 2>Laws of set theory.
- 3>Partitions of set.
- 4>Power set.

TUTORIAL QUESTIONS:-

Q1>State true/false with reasons

- a> $\Phi \subset \{ \Phi \}$
- b> $\{ \} = \{ \{ \}$
- c> Φ subset of Φ
- d> $\{0,0,1\} = \{0,1,1\}$
- e> $\{0\} \in \{0,1,2\}$

Q2>Define following terms . Draw Venn diagrams. Shade the region under consideration.

- a>Proper subset.
- b>Complement of set A with respect to universal set.
- c>Complement of set A with respect to set B.
- d>Symmetric difference.
- e>Partition of a set.

Q3>Find $PP\{\{\}\}$

Q4>Find $((B \cup C) \cap A \cap \Phi)$

Q5> Find $|\{ \Phi \} \cup \{ \{ \Phi \} \}|$

Discrete Structure

Q6> Show order of evaluation using parenthesis $A \cup B^c \cap D \cup E$

Q7> Let Z be a set of integers, N be a set of natural numbers, R is a set of real numbers, C is a set of complex numbers. Show the set containment.

Q8> Find partition of a set containing different letters from word "MISSISSIPPI".

Q9> It is known that at the university 60% of professors play tennis, 50% play bridge, 70% play carrom, 20% play tennis & bridge, 30% play tennis & carrom, 40% play bridge & carrom. If someone claims that 20% of professors play carrom, bridge & tennis, would you believe this claim & why.

Q10> Find symmetric difference between set A & set B . A is a set of vowels in English. B is a set of different letters in sentence "Solving tutorial is a easy task".

TUTORIAL NO 2

TITLE :-SET THEORY AND LOGIC

OBJECTIVE:-To study set theory and logic

REFERENCE:-

1. Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
2. 2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
3. Previous Questions Papers.

PREREQUISITE:-

- 1>Sets, Venn diagram, set membership table
- 2>Laws of set theory.
- 3>Partitions of set.
- 4>Power set.

TUTORIAL QUESTIONS:-

Q1>Write down statement in symbolic form

i>The sun is bright and humidity is not high

ii>If the utility cost goes up or the request for additional funds is denied, then a new computer will be purchased if and only if we can show that the current computing facilities are inadequate.

Q2>Given that value of $p \rightarrow q$ is false, determine value of $(\neg p \vee \neg q) \rightarrow q$.

Q3>If $p \rightarrow q$ is true ,determine $\neg p \vee (p \leftrightarrow q)$.

Q4>Which of the following are statements:

a>Please keep quite and solve your own tutorial.

b>May god bless you with success.

c>The sum of internal angles of a triangle in a plane is two right angles.

d> $4 + 0.6725 = 5.5276$

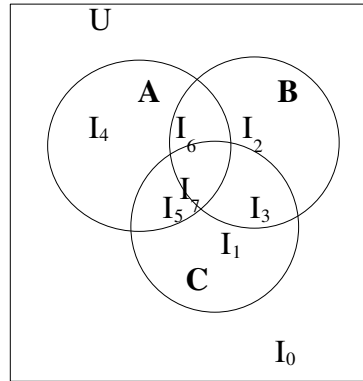
e>This statement is false.

Q5>Using set membership table prove,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Discrete Structure

Q6>



U is universal set ,A,B,C are subsets of U. Show complete intersection of U generated by A,B,C with the help of venn diagram. Given $A \cap B \cap C \neq \emptyset$. Give equations also.

Discrete Structure

TUTORIAL NO:- 3

TITLE:-RELATIONS

OBJECTIVE:-To study relations and properties possessed by relations

REFERENCE:-

1.Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.

2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.

3.Previous Questions Papers.

PREREQUISITE:-

- 1.Relations.
- 2.Path
- 3.Diagraph

TUTORIAL QUESTIONS:-

Q1>Find matrix & diagraph of of relation R on set A
a>A={1,2,3,4,8} aRb if and only if a=b.
b>A={1,2,3,4,8} aRb if and only if a+b<=9

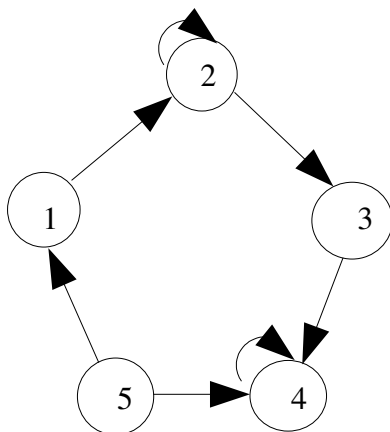
Q2>Let a={1,2,3,4}

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Give digraph.

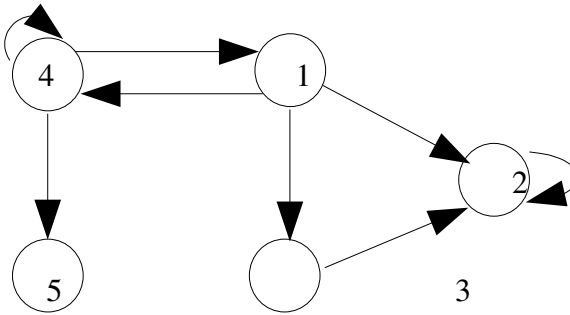
Q3>Find relation determined by

a>

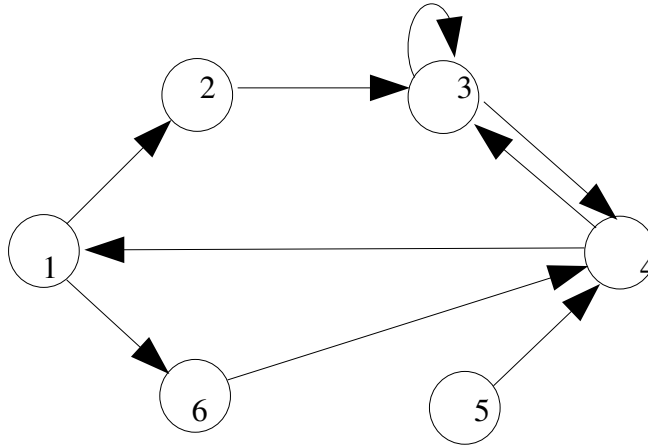


Discrete Structure

b>



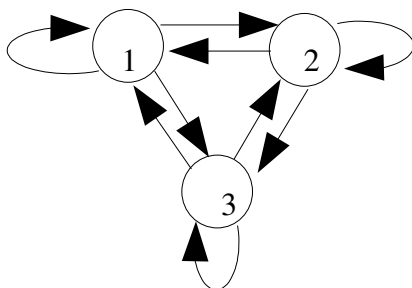
Q4>



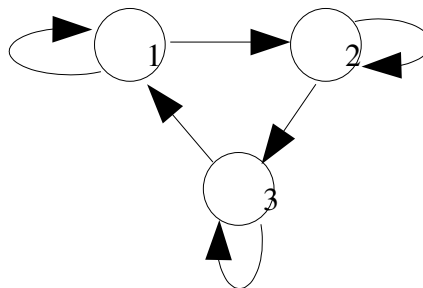
- i> Draw digraph of R^2
- ii> Find M_R^2

Q5> Determine whether relation R whose digraph is given is equivalence

a>

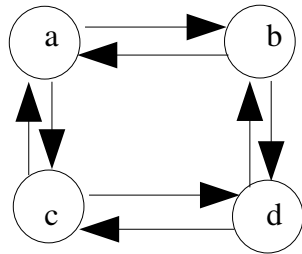


b>



Discrete Structure

c>



Q6> Find partitions corresponding to $R \cap S$

Let $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(1, 2), (1, 1), (2, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (5, 6), (6, 5)\}$

$S = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 3), (3, 2), (4, 6), (4, 4), (6, 4), (6, 6), (5, 5)\}$

TUTORIAL NO :-4

TITLE:- RELATIONS

- OBJECTIVE:-**
- 1.To study Relations,Paths and Diagraphs.
 - 2.To study Manipulation of Relations, Closures,Warshall's Algoritms.
 - 3.To study Posets and Hasse Diagram

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

PRE-REQUISITE:-

- 1.Properties of Relations.
- 2.Reflexive ,Transitive and Symmetric Closures
- 3.Posets (Pratial Order Set).
- 4.Hasse Diagram

TUTORIAL QUESTION:-

Q1> Find Reflexive and Symmetric Closure on the Set A

$$A=\{1,2,3,4,5\}$$
$$R=\{(1,2) (3,4) (2,4) (1,5) \}$$

Q2> Find S^∞

$$M= \begin{vmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{vmatrix}$$

Q3>Draw Hasse Diagram of Relation R on A.

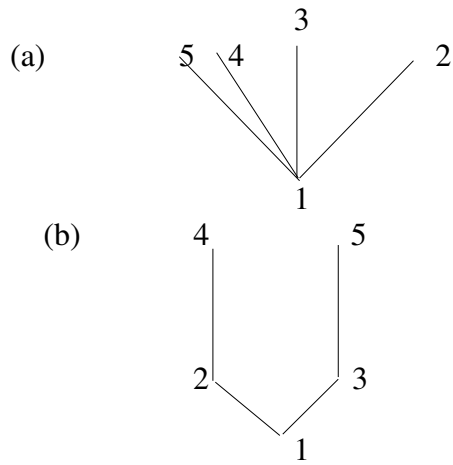
$$A=\{ 1,2,3,4,5\}$$

$$M= \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

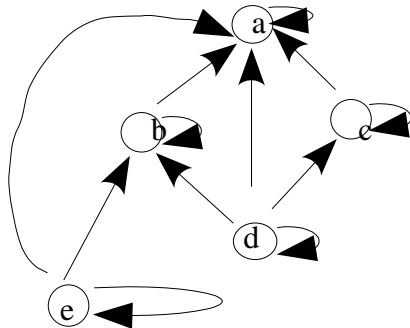
Discrete Structure

0 0 0 0 1

Q4> Determine Matrix of Partial Order whose Hasse Diagram is



Q5> Draw Hasse Diagram of Poset whose Diagram is



TUTORIAL NO :-5

TITLE:-LATTICES

OBJECTIVE:- 1.To study Lattice.

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

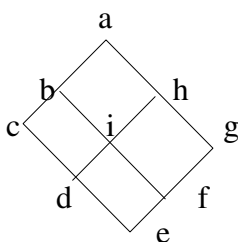
PRE-REQUISITE:-

- 1.Lower Bound
- 2.Upper Bound
- 3.Least Upper Bound
- 4.Greatest Lower Bound
- 5.Posets
- 6.Hasse Diagram
- 7.Lattice

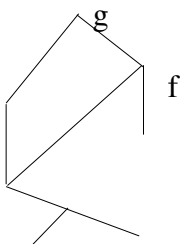
TUTORIALQUESTIONS:-

Q1>Determine whether following diagram are Lattice.

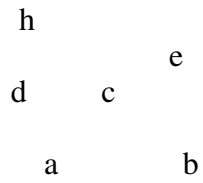
fig(1)



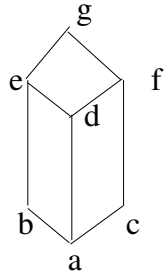
fig(2)



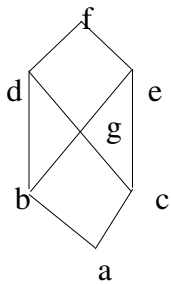
Discrete Structure



fig(3)



fig(4)



Q2> Is Poset (A, \leq) under operation of divisibility a lattice.
 $A = \{2, 3, 6, 12, 24, 36, 72\}$

Q3> If L_1 and L_2 are Lattices, Draw Hasse Diagram for $(L_1 \times L_2)$.

Q4> Find all Sub Lattices of D_{24} containing at least 12 elements.

TUTORIAL NO :-6

TITLE:- TYPES OF LATTICES

OBJECTIVE:- 1.To study Complemented Lattice.
2.To study Distributive Lattice

REFERENCE:-

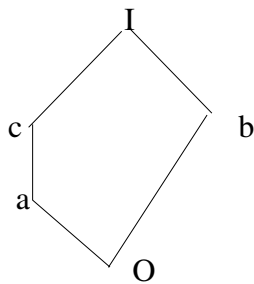
- 1.Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

PRE-REQUISITE:-

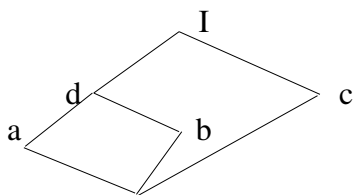
- 1.Complemented Lattice
- 2.Distributive Lattice

TUTORIAL QUESTIONS:-

Q1> Find Complement of element b



Q2> Find Complement of element a



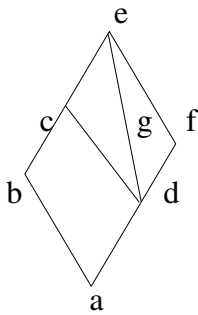
Discrete Structure

o

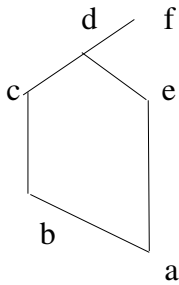
Q3> Find Complement of each element in D_{24} and D_{105} .Are they Complemented Lattices

Q4> Which of the following Lattices are Distributive,complemented or both

fig(1)



fig(2)

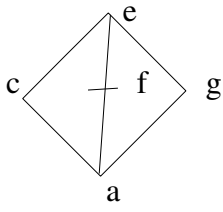


fig(3)



fig(4)

Discrete Structure



TUTORIAL NO:- 7

TITLE :- FUNCTION

OBJECTIVE:-To study functions and properties possessed by function

REFERENCE:-

1. Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
2. 2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
3. Previous Questions Papers.

PREREQUISITE:-

1. Definitions Of
 - a>Function
 - b>Domain
 - c>Codomain
 - d>Range
 - e>Injective
 - f>Surjective
 - g>Bijective
 - h>Composite Functions
 - i>Inverse Function

TUTORIAL QUESTIONS:-

Q1>State which of the following are functions & properties possessed by them

Let $A=\{a_1,a_2,a_3\}$ $B=\{b_1,b_2,b_3\}$ $C=\{c_1,c_2\}$ $D=\{d_1,d_2,d_3,d_4\}$

$f_1 : A \rightarrow B = \{(a_1,b_2),(a_2,b_3),(a_3,b_1)\}$

$f_2 : A \rightarrow D = \{(a_1,d_2),(a_2,d_1),(a_3,d_4)\}$

$f_3 : D \rightarrow B = \{(d_1,b_1),(d_2,b_2),d_3,b_1\}$

Q2>Determine whether f^{-1} exists? Let $f : A \rightarrow B$, $A=\{1,2,3,4\}$, $B=\{a,b,c,d\}$

i> $f = \{(1,a),(2,a),(3,c),(4,d)\}$

ii> $f = \{(1,a),(2,c),(3,b),(4,d)\}$

Q3>Let A be set of real numbers, $B=\{0,1\}$ & Z be set of integers. Let $f : A \rightarrow B$.

Then for any real number a let $f(a)=0$ if $a \notin Z$ & $f(a)=1$ if $a \in Z$ State whether it is function or not?

Q4>Determine whether function is one to one & onto. Let $f : A \rightarrow B$

a>Let set $A \rightarrow B$ =set of integers $f(a)=a-1$.

b>Let set A =set of real numbers. $B=\{x/x \text{ is real \& } x \geq 0\}$ $f(a)=|a|$.

Q5>Let f,g,h be functions from $Z \rightarrow Z$ where Z is set of integers.

Discrete Structure

$f(n)=n+5$ $g(n)=n-2$ $h(n)=n^2$ Find $f \circ g$, f^3 , $f \circ g \circ h$

TUTORIAL 8

TITLE :-GROUP

OBJECTIVE :-To study groups & properties of groups

REFERENCE:-

1. Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
2. 2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
3. Previous Questions Papers.

PREREQUISITE:-

- 1> Monoids, Semigroup, Groups
- 2> Product and quotients of algebraic structures
- 3> Isomorphism, homomorphism, automorphism
- 4> Normal subgroup

TUTORIAL QUESTIONS :-

Q1> Consider binary operation * defined on the set $A = \{a,b,c,d\}$ by the following table

	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Compute

a> $c*d$ & $d*c$

b> $b*d$ & $d*b$

c> $a*(b*c)$ & $(a*b)*c$

d> Is * commutative? Is * associative?

Q2> Let $A=\{a,b\}$ which of the following tables define a semigroup on A? Which define a monoid on A?

	a	b
a	a	b
b	a	a

	a	b
a	a	b
b	b	b

Discrete Structure

	a	b
a	a	a
b	b	b

	a	b
a	b	b
b	a	a

Q3> Let \mathbb{R}^+ be the set of all positive real numbers. Show that the function $f : \mathbb{R}^+ \rightarrow \mathbb{R}$ defined by $f(x) = \ln(x)$ is an isomorphism of semigroup $(\mathbb{R}^+, *)$ to the semigroup $(\mathbb{R}, +)$ where $*$ & $+$ are ordinary multiplication & addition respectively.

Q4> Determine whether the set together with binary operation is a group. If it is a group, determine if it is abelian, specify the identity & inverse.

i> The set of odd integers under operation of multiplication.

ii> \mathbb{Q} , the set of all rational numbers under operation of addition.

Q5> Let G be a group of integers under operation g addition & let $H = \{3 * K / K \in \mathbb{Z}\}$ Where \mathbb{Z} is set of integers. Is H subgroup of G ?

Q6> Prove that function $f(x) = |x|$ is a homomorphism from group G of nonzero real numbers under multiplication to group G' of positive real numbers under multiplication.

Q7> Let Q be set of positive rational numbers which can be expressed in the form $2a^3b$ where a & b are integers. Prove that algebraic structure $(Q, *)$ is a group.

Q8> Let $S = \{x/x \text{ is a real number; } x \neq 0, 1\}$ Consider following functions $f_i : S \rightarrow S$, $i = 1, 2, 3, 4, 5, 6$
 $f_1(x) = x$ $f_2(x) = 1 - x$ $f_3(x) = 1/x$ $f_4(x) = 1/(1-x)$
 $f_5(x) = 1 - (1/x)$ $f_6(x) = x/(x-1)$ Show that $G = \{f_1, f_2, f_3, f_4, f_5, f_6\}$ is group under operation of composition. Give operation table of $(G, *)$.

Q9> Let $G = \{e, a, a^2, a^3, a^4, a^5\}$ be a group under operation $a^i a^j = a^r$ where $i+j=r \pmod{6}$. Prove that G & \mathbb{Z}_6 are isomorphic.

TUTORIAL NO :-9

TITLE:- GRAPHS

OBJECTIVE:- 1.To study graph concepts
2.To study paths and circuits :(Eulerian, Hamiltonian)

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross, Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

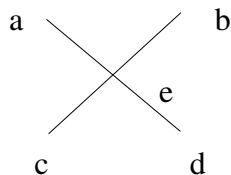
PRE-REQUISITE:-

- 1.Notation and Definition
- 2.Paths and Connectivity
- 3.Types of Graphs
- 4.Subgraphs
- 5.Isomorphic Graphs
- 6.Homomorphic Graphs
- 7.Representation of graphs
- 8.Eulerian and Hamiltonian Graphs

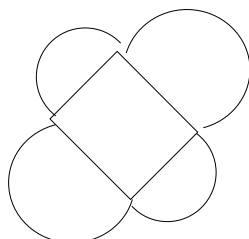
TUTORIAL QUESTION:-

Q1>Determine whether following graphs have Euler's and Hamiltonian path, Circuit.
Give reason.

Fig(1)

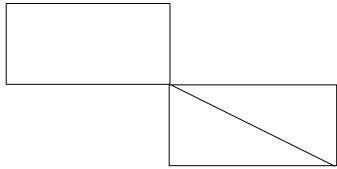


Fig(2)

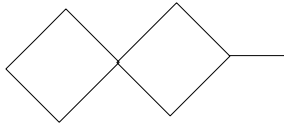


Discrete Structure

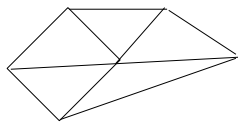
Fig(3)



Fig(4)

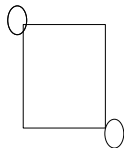


Fig(5)

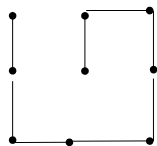


Q2> Verify $\sum_{i=1}^n \deg(V_i) = 2|E|$ if graph is planar

Fig(1)

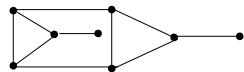


Fig(2)



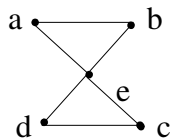
Discrete Structure

Fig(3)

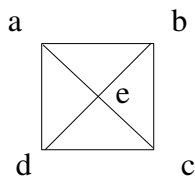


Q3> Which of the undirected graphs in fig's have an Euler circuit ? Write Euler circuit and path in each of the graphs

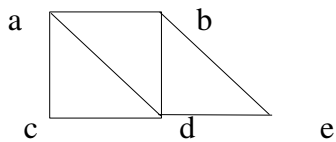
fig(1)



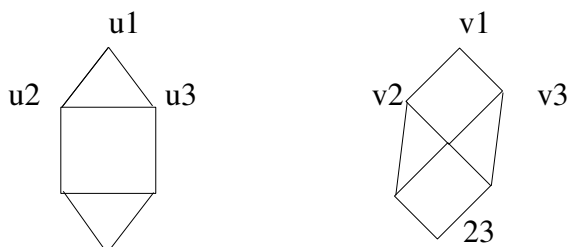
fig(2)



fig(3)



Q4> Determine graph G and H shown in figure are isomorphic or not ? Justify your answer



Discrete Structure

u4 u5 v4 v5
 u6 v6

TUTORIAL NO :-10

TITLE:- RINGS AND FIELDS

OBJECTIVE:-

- 1.To study Rings and Fields concepts
- 2.To study Ring isomorphism and homomorphism

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

PRE-REQUISITE:-

Definitions:-

- 1.Rings
- 2.Fields
- 3.Integral Domain
- 4..Ring Isomorphic

TUTORIAL QUESTION:-

Q1> Let $G=\{e,a,a^2,a^3,a^4,a^5\}$ be a group under operation $a^i a^j = a^r$ where $i+j=r(\text{mod } 6)$
Prove that G & Z_6 are isomorphic.

Q2> Let Z be a group of integers under addition and Let H be a Subgroup of Z consisting of the multiples of 5. Show that H is normal Subgroup of Z and find quotient group Z/H .

Q3> Show that Ring Z_{29} of integers modulo 29 ia an Integral Domain Where Ring Z_{105} of integers modulo 105 is not an Integral Domain

Q4>Let $R=\{ 0,2,4,6,8\}$ Show that R is Commutative Ring under $+$ and \times modulo 10. Verify whether it is an Integral Domain or Field or both

Discrete Structure

Q5> Show that the set $R = \{x \mid x = a + b\sqrt{2}; a, b \text{ are integers}\}$ is a Ring with ordinary addition and multiplication

Q6> Show that $(I, +, \cdot)$ is a commutative Ring identify where the operations $+$ and \cdot are defined as follows:-
 $a + b = a + b - 1$
 $a \cdot b = a + b - ab$

Q7> Let $R = \{0, 2, 4, 6, 8\}$ Show that R is Commutative Ring under addition and multiplication modulo 10.

	0	2	4	6	8
0	0	2	4	6	8
2	2	4	6	8	0
4	4	6	8	0	2
6	6	8	0	2	4
8	8	0	2	4	6

TUTORIAL NO:-11

TITLE:-ENCODING

OBJECTIVE:-To study encoding function.

REFERENCE:-

1. Discrete mathematical structures by Kolman ,Busby and Ross, Fourth Edition.
2. 2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
3. Previous Questions Papers.

PREREQUISITE:-

1. Hamming distance.
2. Group codes.
3. Encoding using parity check matrix.

TUTORIAL QUESTIONS:-

Q1> Find distance between x & y.

$$\begin{array}{ll} \text{a} > x=1100010 & y=1010011 \\ \text{b} > x=0110110 & y=0011010 \end{array}$$

Q2> Consider the (2,6) encoding function $e : B^2 \rightarrow B^6$ defined by

$$\begin{array}{l} e(00)=000000 \\ e(10)=101010 \\ e(01)=011110 \\ e(11)=111000 \end{array}$$

i> Find minimum distance of e.

ii> How many errors will e detect.

Q3> Show that (2,5) encoding function $e : B^2 \rightarrow B^5$ defined by

$$e(00)=00000$$

Discrete Structure

$$e(01)=01110$$

$$e(10)=10101$$

$$e(11)=11011$$

is a group code.

Q4> Let

$$H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be a parity check matrix. Determine the (2,5) group code function $e : B^2 \rightarrow B^5$

Q5> Let

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be a parity check matrix. Determine the (3,6) group code function $e : B^3 \rightarrow B^6$

TUTORIAL NO:- 12

TITLE:-DECODING,PIGEONHOLE PRINCIPLE

OBJECTIVE:-To study maximum likelihood principle,pigeonhole principle,permutation functions.

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross, Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

PREREQUISITE:-

1. Maximum likelihood principle.
2. Pigeon hole principle.
3. Permutation functions.

TUTORIAL QUESTIONS

Q1>Consider (3,6) group encoding function $e : B^3 \rightarrow B^6$ defined by

$e(000)=000000$	$e(100)=1000101$
$e(001)=000110$	$e(101)=100011$
$e(010)=010010$	$e(110)=110111$
$e(011)=010100$	$e(111)=110001$

Decode the following words relative to maximum likelihood decoding function
 $a>011110$ $b>101011$ $c>110010$

Q2>Consider (3,8) encoding function $e:B^3 \rightarrow B^8$ defined by

Discrete Structure

$$e(000)=00000000$$

$$e(001)=10111000$$

$$e(010)=00101101$$

$$e(011)=10010101$$

$$e(100)=10100100$$

$$e(101)=10001001$$

$$e(110)=00011100$$

$$e(111)=00110001$$

& let d be $(8,3)$ maximum likelihood decoding function associated with e . How many errors can (e,d) correct?

Q3> Show that if any 5 numbers from 1 to 8 are chosen, then two of them will add to 9.

Q4> Show that if 4 numbers from 1 to 7 are chosen, then two of them will add to 8.

Q5> Show that at least 2 people out of 13 must have their birthday in same month when they are assembled in the same room.

Q6> Let $A=\{1,2,3\}$ Let $f : A \rightarrow A$. $f(a)=a$.

a> Find all permutations of A .

b> Compute P_4^{-1} Where P_4 is 4th permutation of A .

c> Compute $P_3 \circ P_2$ Where P_3 & P_2 are 3rd & 2nd permutations of A .