

Discrete Structure

TUTORIAL MANUAL

SUBJECT:

DISCRETE STRUCTURE

CLASS:

(Computer / I.T)

Semester III

Discrete Structure

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TUTORIAL NO 1

TITLE :-SET THEORY

OBJECTIVE:-To study set theory.

REFERENCE:-

1. Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
2. 2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
3. Previous Questions Papers.

PREREQUISITE:-

- 1>Sets,Venn diagram,set membership table
- 2>Laws of set theory.
- 3>Partitions of set.
- 4>Power set.

TUTORIAL QUESTIONS:-

Q1>State true/false with reasons

$$E = \{ \{ 1,2,3 \}, \{ 2,3 \}, \{ a,b \} \}, \quad F = \{ \{ a,b \}, \{ 1,2 \} \}$$

a> $\Phi \subset \{ \Phi \}$

b> $\{ \} = \{ \{ \} \}$

c> Φ subset of Φ

d> $\{ a,b \} \subseteq F$

e> $F \subseteq E$

f> $1 \in E$

g> $\emptyset \subseteq F$

Q2>Define following terms . Draw Venn diagrams. Shade the region under consideration.

a>Proper subset.

b>Complement of set A with respect to universal set.

c>Complement of set A with respect to set B.

d>Symmetric difference.

e>Partition of a set.

Q3>Find $PP\{ \{ a,b \} \}$

Q4>Find $((B \cup C) \cap A) \cap \Phi$

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Q5> Find $|\{\Phi\} \cup \{\{\Phi\}\}|$

Q6> Show order of evaluation using parenthesis $A \cup B \cap D^c \cup E^c$

Q7> Let Z be a set of integers, N be a set of natural numbers, R is a set of real numbers, C is a set of complex numbers. Show the set containment.

Q8> Find partition of a set containing different letters from word "INSIDEINDIA".

Q9> The students in a dormitory were asked whether they had a dictionary (D) or a thesaurus (T) in their rooms. The results showed that 650 students had a dictionary, 150 did not have a dictionary, 175 had a thesaurus, and 50 had neither a dictionary nor a thesaurus. Find the number K of students who :-

a> live in the dormitory,

b> have both a dictionary and a thesaurus, and

c> have only a thesaurus

Q10> Find symmetric difference between set A & set B . A is a set of vowels in English. B is a set of different letters in sentence "Discrete paper is easy".

TUTORIAL NO 2

TITLE :-SET THEORY AND LOGIC

OBJECTIVE:-To study set theory and logic

REFERENCE:-

1. Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
2. 2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
3. Previous Questions Papers.

PREREQUISITE:-

- 1>Sets,Venn diagram,set membership table
- 2>Laws of set theory.
- 3>Partitions of set.
- 4>Power set.

TUTORIAL QUESTIONS:-

Q1>Write down statement in symbolic form

Let p be the “sam is rich “ and let q be “sam be happy”. (Assume “sam is poor “ means “sam is not rich “, i.e. , $\sim p$).

- a> sam is poor but happy.
- b>sam is neither rich nor haapy
- c> sam is either rich or unhappy.
- d>sam is poor or else the is rich and unhappy.

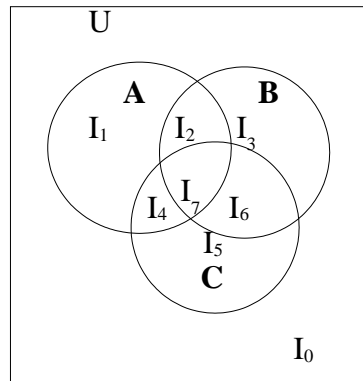
Q2>Given that value of $p \rightarrow q$ is true,determine value of $(\neg p \vee \neg q) \rightarrow q$.

Q3>If $p \rightarrow q$ is false ,determine $\neg p \vee (p \leftrightarrow q)$.

Q4>Construct te truth table of $(p \leftrightarrow q) \leftrightarrow (p \wedge q)$.

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Q6>



U is universal set ,A,B,C are subsets of U. Show complete intersection of U generated by A,B,C with the help of venn diagram. Given $A \cap B \cap C \neq \emptyset$. Give equations also.

TUTORIAL NO:- 3

TITLE:-RELATIONS

OBJECTIVE:-To study relations and properties possessed by relations

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

PREREQUISITE:-

- 1.Relations.
- 2.Path
- 3.Diagraph

TUTORIAL QUESTIONS:-

Q1>Find matrix & diagraph of of relation R on set A

a>A={1,2,3,4,8,10,12} aRb if and only if a=b.

b>A={1,2,3,4,8,9, 10 ,11,12} aRb if and only if a-b>=4

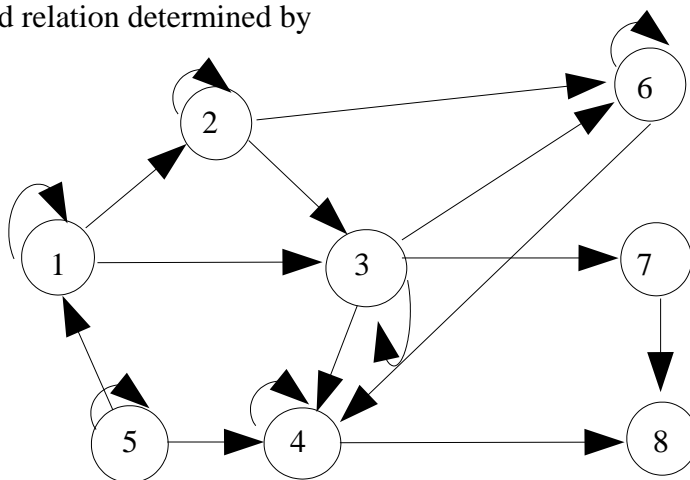
Q2>Let a={1,2,3,4}

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Give digraph.

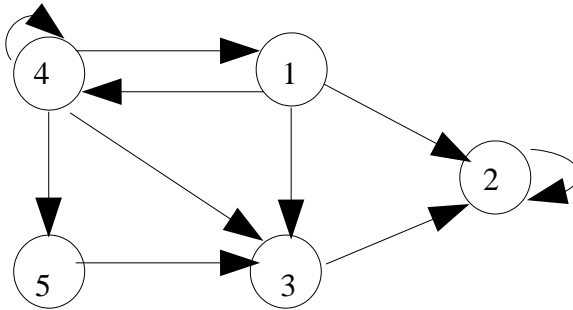
Q3>Find relation determined by

a>

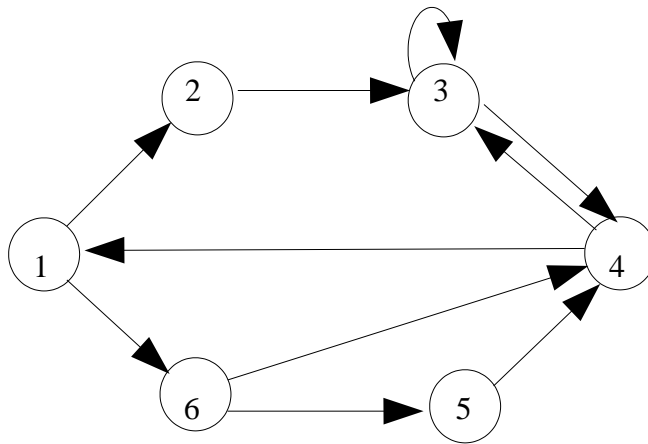


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b>



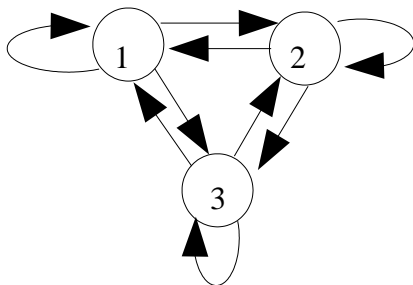
Q4>



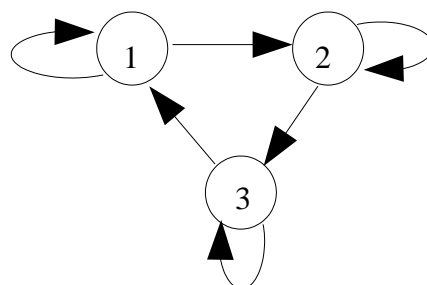
- i> Draw digraph of \mathbb{R}^2
- ii> Find $M_{\mathbb{R}^2}$

Q5> Determine whether relation R whose digraph is given is equivalence

a>

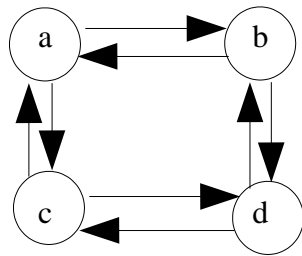


b>



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c>



Q6>Find partitions corresponding to $R \cap S$

Let $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(1, 2), (1, 1), (2, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (5, 6), (6, 5)\}$

$S = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 3), (3, 2), (4, 6), (4, 4), (6, 4), (6, 6), (5, 5)\}$

TUTORIAL NO :-4

TITLE:- RELATIONS

- OBJECTIVE:-**
- 1.To study Relations,Paths and Diagraphs.
 - 2.To study Manipulation of Relations, Closures,Warshall's Algoritms.
 - 3.To study Posets and Hasse Diagram

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

PRE-REQUISITE:-

- 1.Properties of Relations.
- 2.Reflexive ,Transitive and Symmetric Closures
- 3.Posets (Pratial Order Set).
- 4.Hasse Diagram

TUTORIAL QUESTION:-

Q1> Find Reflexive and Symmetric Closure on the Set A

$$A = \{1, 2, 3, 4, 5\}$$
$$R = \{(2, 1) (4, 3) (2, 4) (5, 1) \}$$

Q2> Find S^∞

$$M = \begin{vmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{vmatrix}$$

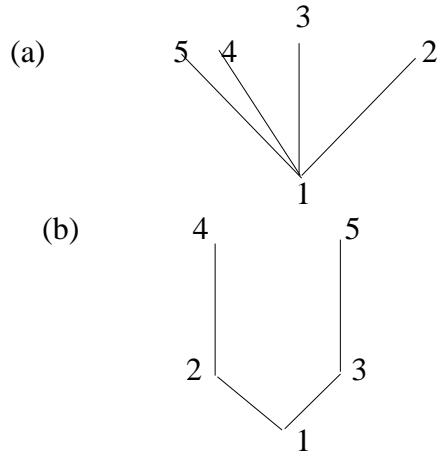
Q3> Draw Hasse Diagram of Relation R on A.

$$A = \{1, 2, 3, 4, 5\}$$

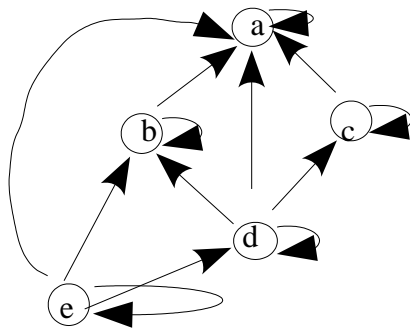
$$M = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{vmatrix}$$

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Q4> Determine Matrix of Partial Order whose Hasse Diagram is



Q5> Draw Hasse Diagram of Poset whose Diagram is



TUTORIAL NO :-5

TITLE:-LATTICES

OBJECTIVE:- 1.To study Lattice.

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

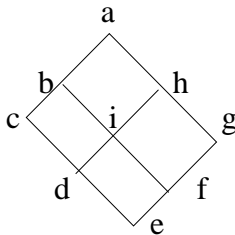
PRE-REQUISITE:-

- 1.Lower Bound
- 2.Upper Bound
- 3.Least Upper Bound
- 4.Greatest Lower Bound
- 5.Posets
- 6.Hasse Diagram
- 7.Lattice

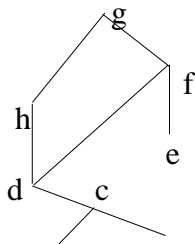
TUTORIALQUESTIONS:-

Q1>Determine whether following diagram are Lattice.

fig(1)



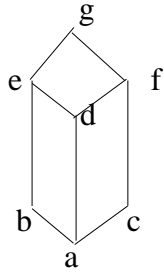
fig(2)



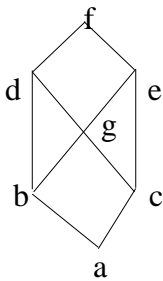
Discrete Structure

a b

fig(3)



fig(4)



Q2> Is Poset (A, \leq) under operation of divisibility a lattice.
 $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$

Q3> If L_1 and L_2 are Lattices, Draw Hasse Diagram for $(L_1 \times L_2)$.

Q4> Find all Sub Lattices of D_{72} containing at least 5 elements.

TUTORIAL NO :-6

TITLE:- TYPES OF LATTICES

OBJECTIVE:- 1.To study Complemented Lattice.
2.To study Distributive Lattice

REFERENCE:-

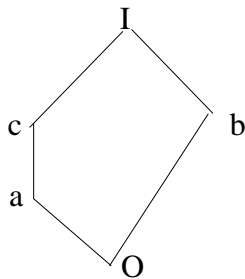
- 1.Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

PRE-REQUISITE:-

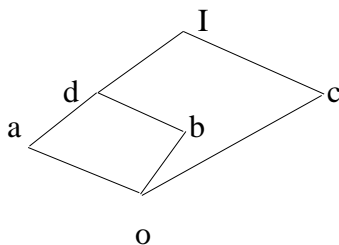
- 1.Complemented Lattice
- 2.Distributive Lattice

TUTORIAL QUESTIONS:-

Q1> Find Complement of element c



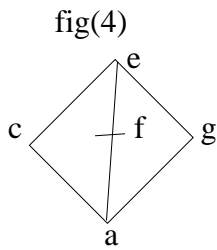
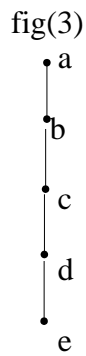
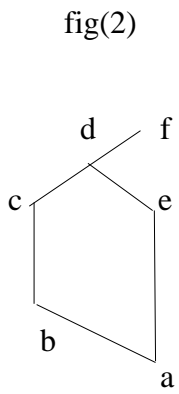
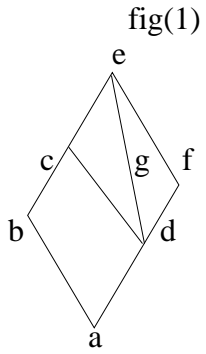
Q2> Find Complement of element b



Q3> Find Complement of each element in D_{60} and D_{105} .Are they Complemented Lattices

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Q4> Which of the following Lattices are Distributive, complemented or both



TUTORIAL NO:- 7

TITLE :- FUNCTION

OBJECTIVE:-To study functions and properties possessed by function

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

PREREQUISITE:-

- 1.Definitions Of
 - a>Function
 - b>Domain
 - c>Codomain
 - d>Range
 - e>Injective
 - f>Surjective
 - g>Bijective
 - h>Composite Functions
 - i>Inverse Function

TUTORIAL QUESTIONS:-

Q1>State which of the following are functions & properties possessed by them

Let $A=\{a_1,a_2,a_3\}$ $B=\{b_1,b_2,b_3\}$ $C=\{c_1,c_2\}$ $D=\{d_1,d_2,d_3,d_4\}$

$f_1 : A \rightarrow B = \{(a_1,b_2),(a_2,b_3),(a_3,b_1)\}$

$f_2 : A \rightarrow D = \{(a_1,d_2),(a_2,d_1),(a_3,d_4)\}$

$f_3 : D \rightarrow B = \{(d_1,b_1),(d_2,b_2),d_3,b_1\}$

Q2>Determine whether f^{-1} exists? Let $f : A \rightarrow B$, $A=\{1,2,3,4\}$, $B=\{a,b,c,d\}$

i> $f = \{(1,a),(2,a),(3,c),(4,d)\}$

ii> $f = \{(1,a),(2,c),(3,b),(4,d)\}$

Q3>Let A be set of real numbers, $B=\{0,1\}$ & Z be set of integers. Let $f : A \rightarrow B$.

Then for any real number a let $f(a)=0$ if $a \notin \mathbb{Z}$ & $f(a)=1$ if $a \in \mathbb{Z}$ State whether it is function or not?

Q4>Determine whether function is one to one & onto. Let $f : A \rightarrow B$

a>Let set $A \rightarrow B =$ set of integers $f(a)=a-1$.

b>Let set $A =$ set of real numbers. $B = \{x/x \text{ is real \& } x \geq 0\}$ $f(a)=|a|$.

Q5>Let f,g,h be functions from $\mathbb{Z} \rightarrow \mathbb{Z}$ where Z is set of integers.

$f(n)=n+5$ $g(n)=n-2$ $h(n)=n^2$ Find fog , f^3 , fogoh

TUTORIAL 8

TITLE :-GROUP

OBJECTIVE :-To study groups & properties of groups

REFERENCE:-

1. Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
2. 2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
3. Previous Questions Papers.

PREREQUISITE:-

- 1> Monoids, Semigroup, Groups
- 2> Product and quotients of algebraic structures
- 3> Isomorphism, homomorphism, automorphism
- 4> Normal subgroup

TUTORIAL QUESTIONS :-

Q1> Consider binary operation * defined on the set $A = \{a,b,c,d\}$ by the following table

	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Compute

a> $c*d$ & $d*c$

b> $b*d$ & $d*b$

c> $a*(b*c)$ & $(a*b)*c$

d> Is * commutative? Is * associative?

Q2> Let $A = \{a,b\}$ which of the following tables define a semigroup on A? Which define a monoid on A?

	a	b
a	a	b
b	a	a

	a	b
a	a	b
b	b	b

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	a	b
a	a	a
b	b	b

	a	b
a	b	b
b	a	a

Q3> Let \mathbb{R}^+ be the set of all positive real numbers. Show that the function $f : \mathbb{R}^+ \rightarrow \mathbb{R}$ defined by $f(x) = \ln(x)$ is an isomorphism of semigroup $(\mathbb{R}^+, *)$ to the semigroup $(\mathbb{R}, +)$ where $*$ & $+$ are ordinary multiplication & addition respectively.

Q4> Determine whether the set together with binary operation is a group. If it is a group, determine if it is abelian, specify the identity & inverse.

i> The set of odd integers under operation of multiplication.

ii> \mathbb{Q} , the set of all rational numbers under operation of addition.

Q5> Let G be a group of integers under operation of addition & let $H = \{3^k \mid k \in \mathbb{Z}\}$ Where \mathbb{Z} is set of integers. Is H subgroup of G ?

Q6> Prove that function $f(x) = |x|$ is a homomorphism from group G of nonzero real numbers under multiplication to group G' of positive real numbers under multiplication.

Q7> Let \mathbb{Q} be set of positive rational numbers which can be expressed in the form $\frac{2a+3b}{a}$ where a & b are integers. Prove that algebraic structure $(\mathbb{Q}, *)$ is a group.

Q8> Let $S = \{x \mid x \text{ is a real number; } x \neq 0, 1\}$ Consider following functions $f_i : S \rightarrow S$, $i = 1, 2, 3, 4, 5, 6$
 $f_1(x) = x$ $f_2(x) = 1-x$ $f_3(x) = 1/x$ $f_4(x) = 1/(1-x)$
 $f_5(x) = 1-(1/x)$ $f_6(x) = x/(x-1)$ Show that $G = \{f_1, f_2, f_3, f_4, f_5, f_6\}$ is group under operation of composition. Give operation table of $(G, *)$.

Q9> Let $G = \{e, a, a^2, a^3, a^4, a^5\}$ be a group under operation $a^i a^j = a^r$ where $i+j=r \pmod{6}$. Prove that G & \mathbb{Z}_6 are isomorphic.

TUTORIAL NO :-9

TITLE:- GRAPHS

OBJECTIVE:- 1.To study graph concepts
2.To study paths and circuits :(Eulerian, Hamiltonian)

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross, Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

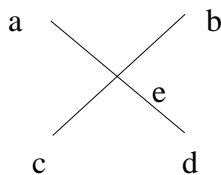
PRE-REQUISITE:-

- 1.Notation and Definition
- 2.Paths and Connectivity
- 3.Types of Graphs
- 4.Subgraphs
- 5.Isomorphic Graphs
- 6.Homomorphic Graphs
- 7.Representation of graphs
- 8.Eulerian and Hamiltonian Graphs

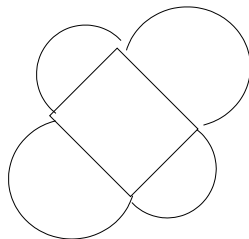
TUTORIAL QUESTION:-

Q1>Determine whether following graphs have Euler's and Hamiltonian path, Circuit.
Give reason.

Fig(1)

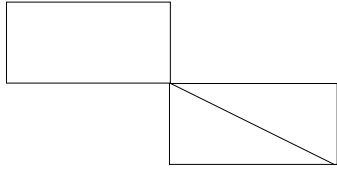


Fig(2)

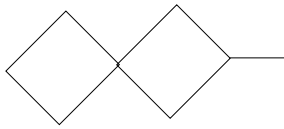


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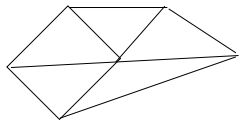
Fig(3)



Fig(4)

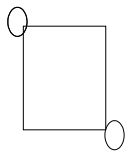


Fig(5)

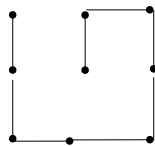


Q2> Verify $\sum_{i=1}^n \deg(V_i) = 2|E|$ if graph is planar

Fig(1)

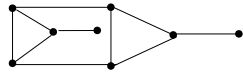


Fig(2)



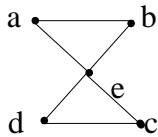
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Fig(3)

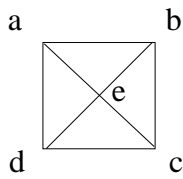


Q3> Which of the undirected graphs in fig's have an Euler circuit ? Write Euler circuit and path in each of the graphs

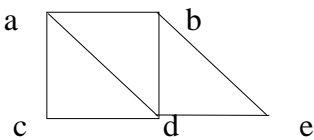
fig(1)



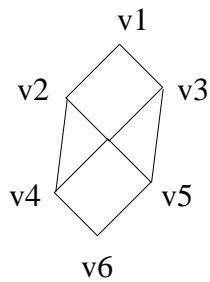
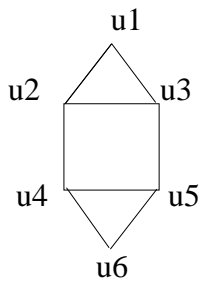
fig(2)



fig(3)



Q4> Determine graph G and H shown in figure are isomorphic or not ? Justify your answer



TUTORIAL NO :-10

TITLE:- RINGS AND FIELDS

OBJECTIVE:-

- 1.To study Rings and Fields concepts
- 2.To study Ring isomorphism and homomorphism

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross,Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

PRE-REQUISITE:-

Definitions:-

- 1.Rings
- 2.Fields
- 3.Integral Domain
- 4..Ring Isomorphic

TUTORIAL QUESTION:-

Q1> Let $G=\{e,a,a^2,a^3,a^4,a^5\}$ be a group under operation $a^i a^j = a^r$ where $i+j=r(\text{mod } 6)$ Prove that G & Z_6 are isomorphic.

Q2> Let Z be a group of integers under addition and Let H be a Subgroup of Z consisting of the multiples of 5. Show that H is normal Subgroup of Z and find quotient group Z/H .

Q3> Show that Ring Z_{29} of integers modulo 29 ia an Integral Domain Where Ring Z_{105} of integers modulo 105 is not an Integral Domain

Q4>Let $R=\{ 0,2,4,6,8\}$ Show that R is Commutative Ring under $+$ and \times modulo 10. Verify whether it is an Integral Domain or Field or both

Q5>Show that the set $R=\{x |x = a+b\sqrt{2} ; a,b \text{ are integers } \}$ is a Ring with ordinary addition and multiplication

Q6>Show that $(I, +, \cdot)$ is a commutative Ring identify where the operations $+$ and \cdot are defined as follows:-
 $a+ b= a+b -1$
 $a \cdot b=a+b-ab$

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Q7> Let $R = \{0, 2, 4, 6, 8\}$ Show that R is Commutative Ring under addition and multiplication modulo 10.

	0	2	4	6	8
0	0	2	4	6	8
2	2	4	6	8	0
4	4	6	8	0	2
6	6	8	0	2	4
8	8	0	2	4	6

TUTORIAL NO:-11

TITLE:-ENCODING

OBJECTIVE:-To study encoding function.

REFERENCE:-

1. Discrete mathematical structures by Kolman ,Busby and Ross, Fourth Edition.
2. 2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
3. Previous Questions Papers.

PREREQUISITE:-

1. Hamming distance.
2. Group codes.
3. Encoding using parity check matrix.

TUTORIAL QUESTIONS:-

Q1> Find distance between x & y.

- a> $x=1100010$ $y=1010011$
b> $x=0110110$ $y=0011010$

Q2> Consider the (2,6) encoding function $e : B^2 \rightarrow B^6$ defined by

- $e(00)=000000$
 $e(10)=101010$
 $e(01)=011110$
 $e(11)=111000$

i> Find minimum distance of e.

ii> How many errors will e detect.

Q3> Show that (2,5) encoding function $e : B^2 \rightarrow B^5$ defined by

- $e(00)=00000$
 $e(01)=01110$
 $e(10)=10101$
 $e(11)=11011$

is a group code.

Discrete Structure

Q4> Let

$$H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be a parity check matrix. Determine the (2,5) group code function $e : B^2 \rightarrow B^5$

Q5> Let

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

be a parity check matrix. Determine the (3,6) group code function $e : B^3 \rightarrow B^6$

TUTORIAL NO:- 12

TITLE:-DECODING,PIGEONHOLE PRINCIPLE

OBJECTIVE:-To study maximum likelihood principle,pigeonhole principle, permutation functions.

REFERENCE:-

- 1.Discrete mathematical structures by Kolman ,Busby and Ross, Fourth Edition.
- 2.2000 Solved problems in discrete mathematics ,Schaum's series , McGraw Hill publication , International edition 1992.
- 3.Previous Questions Papers.

PREREQUISITE:-

1. Maximum likelihood principle.
2. Pigeon hole principle.
3. Permutation functions.

TUTORIAL QUESTIONS

Q1>Consider (3,6) group encoding function $e : B^3 \rightarrow B^6$ defined by

$e(000)=000000$	$e(100)=1000101$
$e(001)=000110$	$e(101)=100011$
$e(010)=010010$	$e(110)=110111$
$e(011)=010100$	$e(111)=110001$

Decode the following words relative to maximum likelihood decoding function
a>011110 b>101011 c>110010

Q2>Consider (3,8) encoding function $e: B^3 \rightarrow B^8$ defined by

$e(000)=00000000$	$e(100)=10100100$
$e(001)=10111000$	$e(101)=10001001$
$e(010)=00101101$	$e(110)=00011100$
$e(011)=10010101$	$e(111)=00110001$

& let d be (8,3) maximum likelihood decoding function associated with e. How many errors can (e,d) correct?

Q3>Show that if any 5 numbers from 1 to 8 are chosen ,then two of them will add to 9.

Discrete Structure

Q4> Show that if 4 numbers from 1 to 7 are chosen , then two of them will add to 8.

Q5> Show that at least 2 people out of 13 must have their birthday in same month when they are assembled in the same room.

Q6> Let $A=\{1,2,3\}$ Let $f : A \rightarrow A .f(a)=a.$

a>Find all permutations of A.

b>Compute P_4^{-1} Where P_4 is 4th permutation of A.

c>Compute $P_3 \circ P_2$ Where P_3 & P_2 are 3rd & 2nd permutations of A.